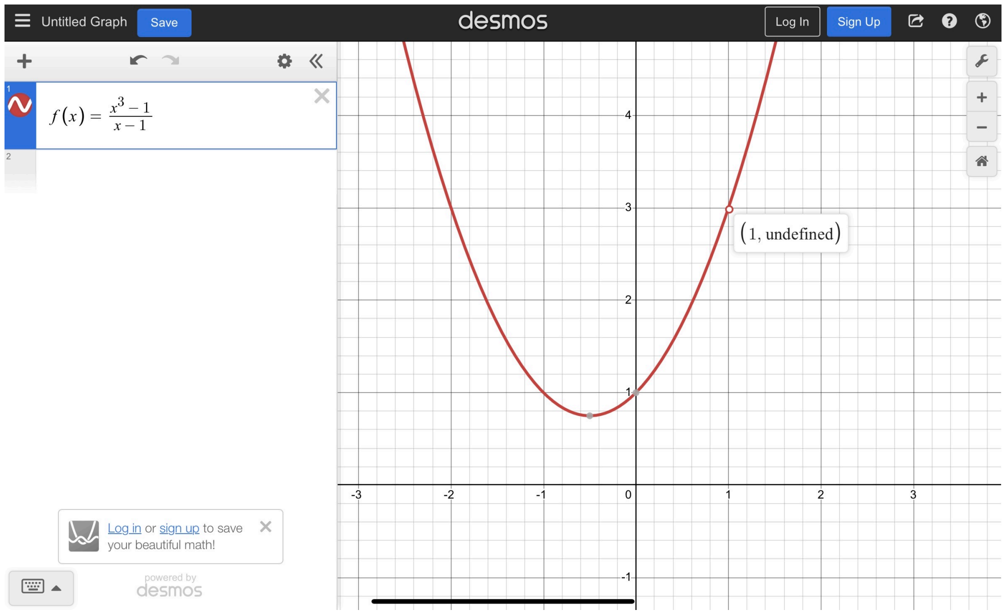
The limit of a function:

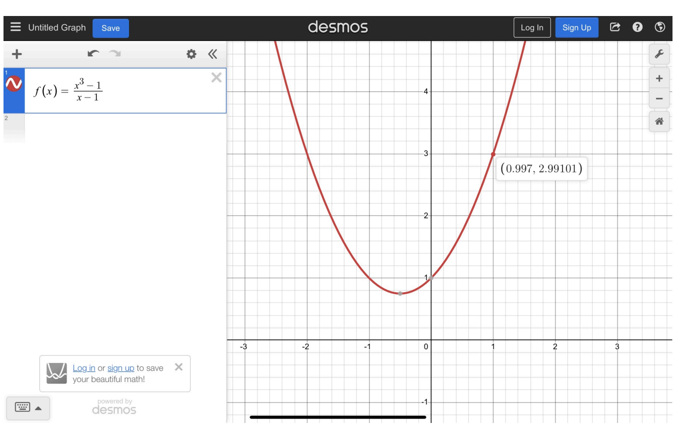
The graph of $f(x) = \frac{x^3-1}{x-1}$!!

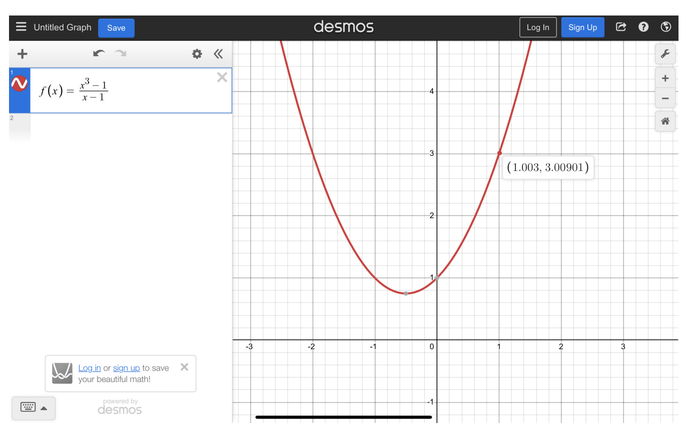


As per desmos, $f(1)$ is Undefined.

Why!! If we just substitute $x=1$ in the given function we get $f(1) = \frac{1^3-1}{1-1} = \frac{0}{0}$ ⟵ Undefined

But what happens around that point where it is undefined!!





<u>Observation</u> : Although $f(1)$ is undefined, but if you move arbitrarily close to 1, $f(x)$ also goes arbitrarily closed to 3

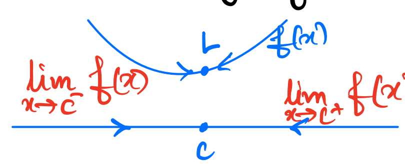
So we can say: as x approces to 1 from left or right, $f(x)$ becomes arbitrarily close to 3

In Math notations we write,

$$\lim_{x \to 1} f(x) = 3 \iff \lim_{x \to 1^-} f(x) = 3 \ \& \ \lim_{x \to 1^+} f(x) = 3$$

Definition of Limit: If $f(x)$ becomes arbitrarily close to a fixed number L as x approaches to c from left & right, then we say the limit of f exists at c & write $\lim_{x \to c} f(x) = L$

$\uparrow$ limiting value



<u>Note</u>: ① It can happen that $f(c)$ & $\lim_{x \to c} f(x)$ have the same value. This situation is Special.

② It can also happen that if we approach to c from left & from right, & get two different values of $f(x)$, mathematically,

$$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$$

this phenomenon is very common with piecewise functions & will be studying soon.

$$\Downarrow$$

$$\lim_{x \to c} f(x) = \text{DNE}$$

Example: Find $\lim_{x \to 0} \frac{\sin x}{x}$.

Here, $f(x) = \frac{\sin x}{x}$

x	$f(x)$	x	$f(x)$
-0.1	0.9983341	0.1	0.9983341
-0.01	0.99998333	0.01	0.99998333
-0.001	0.999999833	0.001	0.999999833
-0.0001	0.999999999	0.0001	0.999999999

x is approaching 0 from left — almost 1

$$\Downarrow$$

$$\lim_{x \to 0^-} f(x) = 1$$

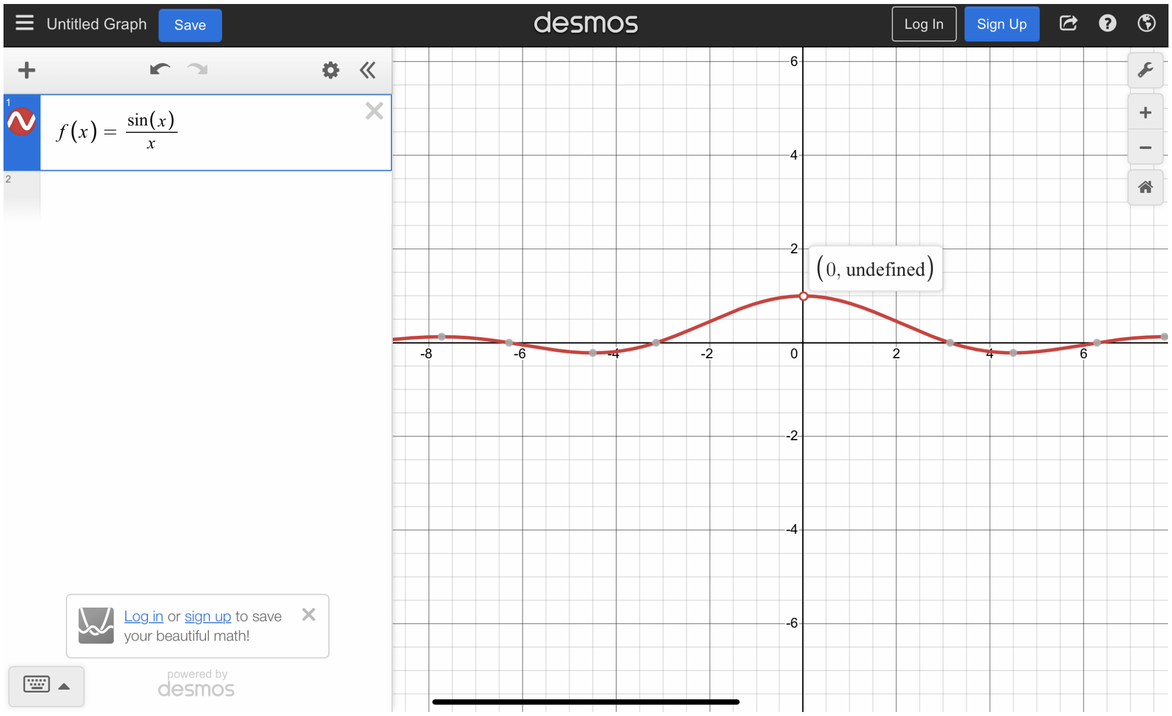
x is approaching 0 from right — almost 1

$$\Downarrow$$

$$\lim_{x \to 0^+} f(x) = 1$$

$$\lim_{x \to 0} f(x) = 1$$

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$



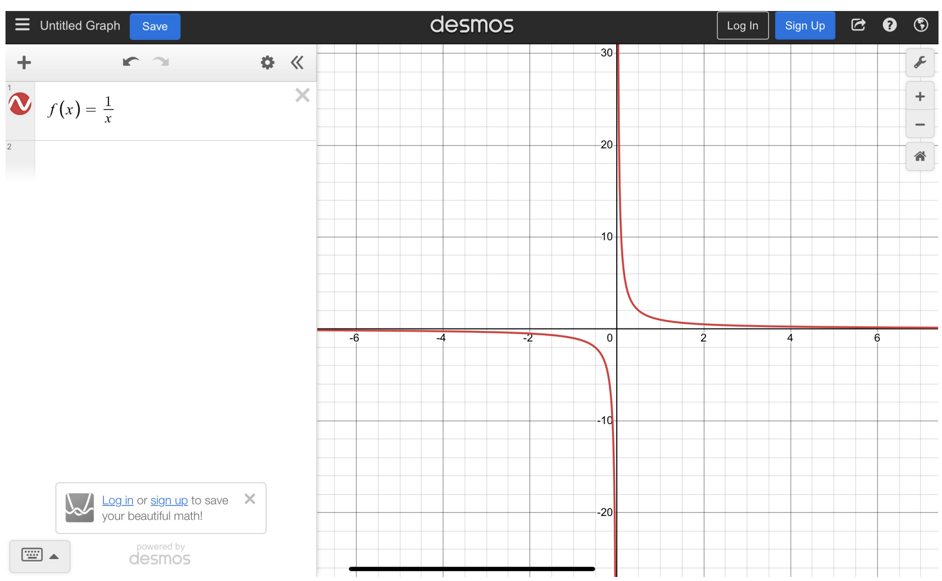
Example: $\lim_{x \to 0} \frac{1}{x}$, find the limit if exists.

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$

$\Downarrow$

$\lim_{x \to 0} \frac{1}{x} = \text{DNE}$



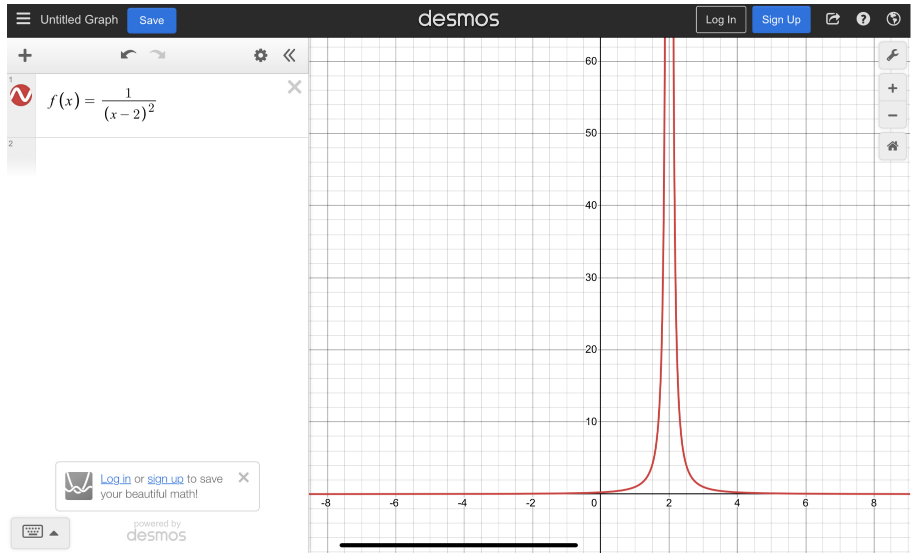
Example: Find $\lim\limits_{x \to 2} \frac{1}{(x-2)^2}$

$$\lim_{x \to 2^-} \frac{1}{(x-2)^2} = +\infty$$

$$\lim_{x \to 2^+} \frac{1}{(x-2)^2} = +\infty$$

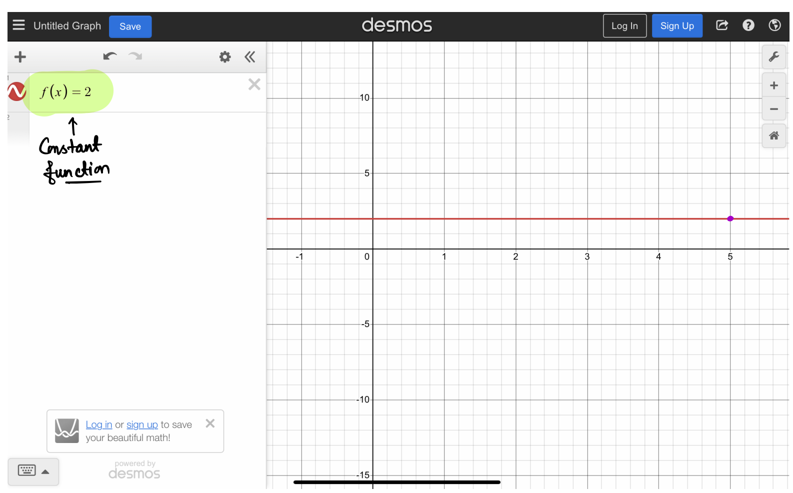
$$\Downarrow$$

$$\lim_{x \to 2} \frac{1}{(x-2)^2} = +\infty$$



Example: Find $\lim\limits_{x \to 5} (2)$

$$\left.\begin{aligned} \lim_{x \to 5^-} (2) &= 2 \\ \lim_{x \to 5^+} (2) &= 2 \end{aligned}\right] \Rightarrow \lim_{x \to 5} 2 = 2$$



Note: $\lim\limits_{x \to a} (b) = b$

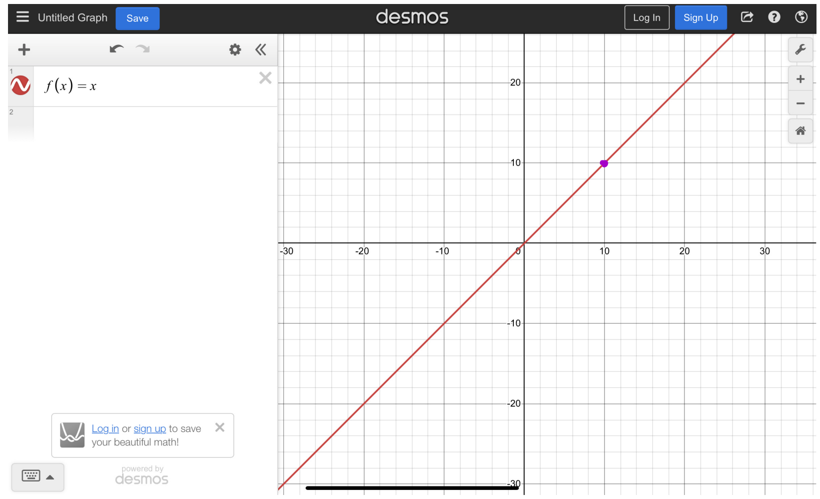
Example: Find $\lim_{x \to 10} f(x)$, where $f(x) = x$, for all $x \in \mathbb{R}$

$$\lim_{x \to 10^-} f(x) = \lim_{x \to 10^-} x = 10$$

$$\lim_{x \to 10^+} f(x) = \lim_{x \to 10^+} x = 10$$

$$\Downarrow$$

$$\lim_{x \to 10} f(x) = \lim_{x \to 10} (x) = 10$$



Note: ① $\lim_{x \to a} x = a$

② $\lim_{x \to a} x^n = a^n$, for all $n > 0$.

③ $\lim_{x \to a} p(x) = p(a)$, where $p(x)$ is a polynomial in x.